Abstract—This paper investigates a wireless powered communication network (WPCN) under the protocol of harvest-then-transmit, where a hybrid access point with constant power supply replenishes the passive user nodes by wireless power transfer in the downlink, then each user node transmit independent information to the hybrid AP in a time division multiple access (TDMA) scheme in the uplink. The sum-throughput maximization and min-throughput maximization problems are considered in this paper. The optimal time allocation for the sum-throughput maximization is proposed based on the Jensen’s inequality, which provides more insight into the design of WPCNs. A low-complexity fixed-point iteration algorithm for the min-throughput maximization problem, which promises a much better computation complexity than the state-of-the-art algorithm. Simulation results confirm the effectiveness of the proposed algorithm.

I. INTRODUCTION

Energy constrained wireless communication networks need some external charging mechanism to replenish their energy and remain active [1]. However, replacing or recharging batteries incurs a high cost and can be inconvenient or highly undesirable (e.g., for sensors embedded in building structures [2] or inside the human body [3]). Harvesting energy from the radio-frequency (RF) signals serves as a safe and promising option, which can be realized by wireless power transfer (WPT) [4]. A reasonable WPT efficiency can be achieved by the state-of-the-art energy harvesting circuits [5]. Moreover, the WPT technology has actually found its application in the wireless sensor networks (WSNs). The interested readers are referred to [6], [7] and the references therein.

It is important to well design the wireless powered communication networks (WPCNs) such that the wireless nodes can be efficiently powered. To this end, the optimal design of WPCNs has drawn a lot of attention in recent years. Since the power and information can be drawn from the radio signals at the same time, some research works were carried out on the hot topic of simultaneous wireless information and power transfer (SWIPT), e.g., [4], [8]–[12]. It was shown that there exist nontrivial tradeoffs between the WPT and wireless information transmission (WIT) efficiency under the single-input single-out (SISO) flat fading channel [8], and the frequency-selective fading channel [9], as well as the multi-input multi-output flat fading channel [4]. The optimal beamforming designs for SWIPT were studied under the two-way relay channel [11] and the MISO interference channel [10], [12]. Besides, it is also important to improve the WPCN system performance from the network perspective [13]–[16].

In this paper, we reconsider the scenario in [16], where a harvest-then-transmit protocol was proposed for a point-to-multipoint network and two system utilities were taken into account, i.e., the sum-throughput maximization and min throughput. Motivated by the fact that the circuit power dissipation is not negligible especially for the WPT based nodes [17], it is hence very important to develop fast and simple enough algorithms for WPCNs. The contribution of this paper is to propose a low-complexity algorithm for the optimal time allocation of the system in [16], such that less energy harvested by WPT will be required for the the baseband processing. This work also provides more insight into the optimal time allocation for the system considered in [16].

This rest of the paper is organized as follows. The system model is described in Sec. II. Then in Sec. III we propose two fast optimal time allocation algorithms for the sum-throughput maximization and the min-throughput maximization, respectively. Sec. IV presents the simulation results for comparison with the algorithm in [16]. Finally, Sec. V concludes the paper.

II. SYSTEM MODEL

This paper considers a SISO WPCN scenario [16], as shown in Fig. 1, which consists of a single-antenna hybrid AP (HAP) and $K$ single-antenna user nodes, denoted by $U_i$ for $i = 1, \ldots, K$. The network operates in a time division multiple access (TDMA) fashion. Assume that the frame duration is normalized to be 1. At the first $\tau_0 \in [0, 1]$ fraction of time, the HAP transfers the power to the $K$ user nodes in the downlink (DL). Then in the uplink (UL), $U_i$ sends its independent information to the HAP within $\tau_i \in [0, 1]$ fraction of time by the energy harvested at the initial slot, for $i = 1, \ldots, K$. Here, the total time constraint reads

$$\sum_{i=0}^{K} \tau_i \leq 1.$$  \hspace{1cm} (1)

Assume that both DL and UL channels, denoted respectively by $\hat{h}_i, \tilde{g}_t \in \mathbb{C}$, are quasi-static flat-fading. The HAP has perfect knowledge of all channel state information. During the DL
WPT phase, the transmitted WPT signal from the HAP is denoted by $x_0 \in \mathbb{C}$, which is subject to the average power limit, i.e., $E[|x_0|^2] \leq P_{\text{max}}$. Thus, the energy harvested at $U_i$ can be expressed by

$$E_i = \tau_0 \xi_i E[|\tilde{h}_i x_0|^2] \leq \tau_0 \xi_i h_{\text{max}},$$

where $\xi_i \in (0, 1)$ is the energy harvesting efficiency at $U_i$, and $\tilde{h}_i \triangleq |h_i|^2$, for $i = 1, \ldots, K$.

In the subsequent UL WIT phase, all nodes transmit their independent information to the HAP with the energy harvested in the DL WPT phase, assuming that all the energy harvested is used. Let the WIT signal transmitted by $U_i$ be $x_i \sim \mathcal{CN}(0, P_i)$. Then its average power is limited by

$$P_i = E[|x_i|^2] = \frac{E_i}{\tau_i}, i = 1, \ldots, K,$$

and the received signal $y_i$ at the HAP in the $i$th UL slot is

$$y_i = \tilde{g}_i x_i + z_i, i = 1, \ldots, K,$$

where $z_i \sim \mathcal{CN}(0, \sigma_i^2)$ represents the additive Gaussian noise at the HAP. Therefore, the achievable throughput of $U_i$ in bits/second/Hz (bps/Hz) can be expressed as

$$R_i(\tau_i, \gamma_i) = \gamma_i \log_2 \left( 1 + \frac{g_i P_i}{\Gamma} \right) \leq \tau_i \log_2 \left( 1 + \frac{\gamma_i \tau_0}{\tau_i} \right),$$

where $g_i \triangleq |\tilde{g}_i|^2$, $\gamma_i = \frac{\xi_i h_i P_{\text{max}}}{\tau_i}$, for $i = 1, \ldots, K$, $\tau \triangleq [\tau_0, \ldots, \tau_K]^T$ and $\Gamma$ denotes the signal-to-noise ratio (SNR) gap due to a practical modulation and coding scheme used.

### III. Problem Formulation

#### A. Time Allocation for Sum-Throughput Maximization

In this subsection, let us focus on the optimal time allocation with the objective of sum-throughput maximization. From (5), the problem can be formulated as

$$\max_{\tau} \sum_{i=1}^{K} R_i(\tau_0, \tau_i)$$

s.t.

$$R_i(\tau_0, \tau_i) \leq \tau_i \log_2 \left( 1 + \frac{\gamma_i \tau_0}{\tau_i} \right), \forall i = 1, \ldots, K,$$

$$\tau_i \geq 0, \forall i = 1, \ldots, K,$$

$$\sum_{i=0}^{K} \tau_i \leq 1.$$
B. Time Allocation for Min-Throughput Maximization

In the previous subsection, the same SNR is achieved at the optimal solution. In view of the user fairness, the following min-throughput maximization problem is considered

$$\begin{align*}
\max_{\tau_i \in [0, 1], i = 1, \ldots, K} & \min_{i = 1, \ldots, K} R_i (\tau_0, \tau_i) \\
\text{s.t.} & \quad 0 \leq \tau_i \leq 1, \forall i = 0, \ldots, K, \quad \sum_{i=0}^{K} \tau_i \leq 1. \tag{12a}
\end{align*}$$

For convenience, we redefine $\tau \triangleq [\tau_1, \ldots, \tau_K]^T$ and denote $T(\tau_0) \triangleq \{ \tau_i | \tau_i \in [0, 1], \forall i = 1, \ldots, K, \quad 1^T \tau \leq 1 - \tau_0 \}$. Thus the problem (12) can be alternatively expressed as

$$\begin{align*}
\max_{\tau_0 \in [0, 1]} & \quad g(\tau_0) \triangleq \max_{\tau \in T(\tau_0)} \bar{g}(\tau_0, \tau), \tag{13}
\end{align*}$$

where $\bar{g}(\tau_0, \tau) = \min_{i = 1, \ldots, K} R_i (\tau_0, \tau_i)$. It can be observed that $g(0) = g(1) = 0$. However, what we are interested in is to show the following lemma:

**Proposition 1** $g(\tau_0)$ is strictly concave w.r.t. $\tau_0 \in [0, 1]$. **Proof:** Please refer to the Appendix.

Now let us focus on the time allocation for UL WITs with any given $\tau_0 \in [0, 1]$. By introducing a slack variable $\bar{R}$, the problem associated with $g(\tau_0)$ is equivalent to

$$\begin{align*}
\max_{\tau, R} & \quad \bar{R} \\
\text{s.t.} & \quad \tau_i \log_2 \left( 1 + \frac{\gamma_i \tau_i}{\bar{R}} \right) \geq \bar{R}, \quad \forall i = 1, \ldots, K, \tag{14a}
\tau_i \in [0, 1], \quad \forall i = 1, \ldots, K, \tag{14b}
\quad 1^T \tau \leq 1 - \tau_0, \tag{14c}
\end{align*}$$

which is convex and can thus be solved by, e.g., the subgradient approach in [16].

But we are more interested in a fast fixed-point iteration algorithm in this paper. To this end, we notice that the throughput constraints in (14b) and the time constraint (14d) are active at the optimum, that can be easily verified by contradiction. Besides, the problem (14) can be solved by bisection search over $\bar{R}$, and thereby checking the feasibility for the given $\bar{R}$, which is given by

$$\begin{align*}
\text{find } & \quad \tau \\
\text{s.t.} & \quad \tau_i \log_2 \left( 1 + \frac{\gamma_i \tau_i}{\bar{R}} \right) = \bar{R}, \quad \forall i = 1, \ldots, K, \tag{15a}
\tau_i \in [0, 1], \quad \forall i = 1, \ldots, K. \tag{15b}
\end{align*}$$

**Lemma 2** For any given $\tau_0$ and $\bar{R}$, (15b) gives rise to the unique solution for $\tau_i^*$, which can be obtained by the following fixed-point iteration with linear convergence rate:

$$\begin{align*}
\tau_i[n] &= \frac{\bar{R}}{\log_2 \left( 1 + \frac{\gamma_i \tau_i[n]}{\bar{R}} \right)}, \quad \forall i = 1, \ldots, K, \tag{16}
\end{align*}$$

where $n$ is the iteration index. **Proof:** It can be proved that the function $f(x) = \frac{\bar{R}}{\log_2 \left( 1 + \frac{\gamma_i \tau_i[n]}{\bar{R}} \right)}$ is Lipschitz continuous with Lipschitz constant $L < 1$, then $f(x)$ has precisely one fixed point. The detailed proof is omitted due to space limitations. \hfill \blacksquare

From Lemma 2, the feasibility problem (15) is equivalent to check whether $\tau^* \in T(\tau_0)$ or not, where $\tau^* = [\tau_1^*, \ldots, \tau_K^*]^T$. Furthermore, the optimal WPT time fraction $\tau_0$ can be obtained uniquely by golden section search based on the problem (13). The global optimum can be attained due to the strict concavity of $g(\tau_0)$ as stated in Lemma 1.

To summarize, a fast time allocation algorithm for the problem (12) is detailed in Algorithm 1.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we will show the performance of the proposed algorithm for (12) and compare it with the algorithm in [16]. The parameters are taken from [16], where the channel power gains are modeled as $h_i = g_i = 10^{-3} \rho_i^2 d_i^{-2}$ with $\rho_i$ being the standard Rayleigh fading and $d_i$ being the distance between the HAP and $U_i$, while $\sigma_i^2 = -100$ dBm and $\xi_i = 0.5$, for $i = 1, \ldots, K$. Let the SNR gap $\Gamma = 9.8$ dB. The same throughput accuracy $10^{-3}$ is adopted in the simulations.

Fig. 2 shows the average iteration time vs. the number of WIT nodes, where $d_i$ is uniformly distributed over $[5, 20]$, $P_{\text{max}} = 5$ dBm and the results are averaged over 1000 random realizations. It turns out that the proposed algorithm is much faster than the algorithm in [16], especially when the number of WIT nodes increases.

In order to demonstrate the user fairness, in Fig. 3 we show the individual throughput vs. the power limit $P_{\text{max}}$ with $K = 2$.

**Algorithm 1** A fast time allocation for the problem (12)

1: Initialize $\tau_0, \ldots, \tau_K$, and $\epsilon > 0$;
2: repeat
3: Initialize $R_{\text{min}} = 0$, $R_{\text{max}} > \bar{R}$;
4: repeat
5: $\bar{R} = \frac{1}{2} (R_{\text{min}} + R_{\text{max}})$;
6: Compute $\{ \tau_i \}_i$ by (16);
7: if $\sum_{i=1}^{K} \tau_i > 1 - \tau_0 + \epsilon$ then
8: $R_{\text{max}} \leftarrow R_i$;
9: else if $\sum_{i=1}^{K} \tau_i < 1 - \tau_0 + \epsilon$ then
10: $R_{\text{min}} \leftarrow R_i$;
11: end if
12: until $|1 - \tau_0 - \sum_{i=1}^{K} \tau_i^*| \leq \epsilon$;
13: Update $\tau_0$ by golden section search;
14: until $\tau_0$ converges.
WIT nodes. The results are averaged over 1000 realizations with fixed \(d_1 = 5\) m and \(d_2 = 10\) m. It can be seen that equal throughput is achieved by the proposed algorithm as expected. But there exists a throughput deviation between \(R_1\) and \(R_2\) by the algorithm in [16], which is contradicted with the previous analysis and may be caused by the iteration accuracy of the ellipsoid method in [16].

V. CONCLUSION

In this paper we consider the time allocation for wireless powered communication networks. An optimal time allocation algorithm is proposed for the summ-throughput maximization problem based on the Jensen’s inequality. Then we propose a low-complexity algorithm for the min-throughput maximization problem, which promises a much better computation complexity than the state-of-the-art algorithm. Simulation results confirm the effectiveness of the proposed algorithm.

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APPENDIX

Proof of Lemma 1: Firstly, due to the fact that \(R_i(\tau_0, \tau_i)\) is strictly concave w.r.t. \(\{\tau_0, \tau_i\}\), the pointwise minimum function \(\bar{g}(\tau_0, \tau)\) is also strictly concave w.r.t. \(\{\tau_0, \tau\}\).

Secondly, we will prove the strict concavity of \(g(\tau_0)\) over \([0, 1]\). To show this, we need to prove that for any \(t_1 \in [0, 1]\), \(t_2 \in [0, 1]\), and \(\theta \in (0, 1)\), it admit

\[
g(\theta t_1 + (1 - \theta) t_2) > \theta g(t_1) + (1 - \theta) g(t_2).
\]  

(17)

To this end, we first note that for any feasible points \(t_1\) and \(t_2\) (\(t_1 \neq t_2\)), there exist \(x_1 \in T(t_1)\) and \(x_2 \in T(t_2)\) with

\[
g(t_1) = \bar{g}(t_1, x_1), \quad g(t_2) = \bar{g}(t_2, x_2).
\]  

(18)

Note that \(x_j \in T(t_j)\) implies \(x_j \in [0, 1]\) and \(1^T x_j \leq 1 - t_j\) for \(j = 1, 2\). Hence, for any \(\theta \in [0, 1]\), the point \(\theta x_1 + (1 - \theta) x_2\) satisfies

\[
\theta x_1 + (1 - \theta) x_2 \in \theta[0, 1] + (1 - \theta)[0, 1] = [0, 1] - t_1 - t_2,
\]  

(19a)

\[
1^T (\theta x_1 + (1 - \theta) x_2) = \theta 1^T x_1 + (1 - \theta) 1^T x_2 \leq \theta (1 - t_1) + (1 - \theta) (1 - t_2),
\]  

(19b)

which shows that the point \(\theta x_1 + (1 - \theta) x_2 \in T(\theta t_1 + (1 - \theta) t_2)\).

Therefore, we have

\[
g(\theta t_1 + (1 - \theta) t_2) = \max_{x \in T} \bar{g}(\theta t_1 + (1 - \theta) t_2, x) \geq \bar{g}(\theta t_1 + (1 - \theta) t_2, x_1 + (1 - \theta) x_2) \geq \theta g(t_1) + (1 - \theta) g(t_2),
\]  

(20a)

where the second inequality is due to the strict concavity of \(\bar{g}(\tau_0, \tau)\).

This establishes the strict concavity of \(g(\tau_0)\).

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